AQA

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

I declare this is my own work.

AS MATHEMATICS

Paper 2

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Express as a single power of a

$$\frac{a^2}{\sqrt{a}}$$

where $a \neq 0$

Circle your answer.

[1 mark]

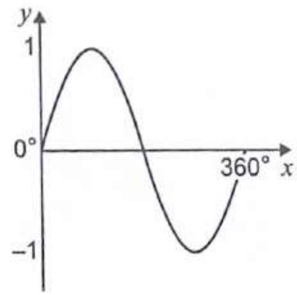
a^1 $\quad$ $a^{\frac{3}{2}}$ $\quad$ $a^{\frac{5}{2}}$ $\quad$ a^4

2 One of the diagrams below shows the graph of $y = \sin(x + 90°)$ for $0° \leq x \leq 360°$

Identify the correct graph.

Tick (✓) **one** box.

[1 mark]



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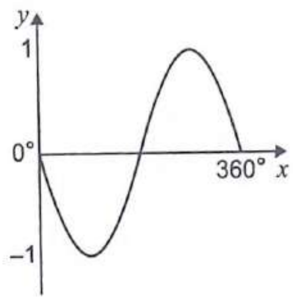

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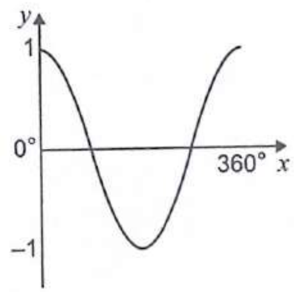

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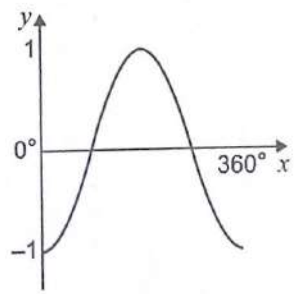

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3 It is given that

$$\frac{dy}{dx} = \sqrt{x}$$

Find an expression for y.

[3 marks]

$$\frac{dy}{dx} = x^{1/2}$$

$$y = \frac{2}{3}x^{3/2} + c$$

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4 (a) Find the binomial expansion of $(1-2x)^5$ in ascending powers of x up to and including the term in x^2

[2 marks]

$1 + 5(-2x) + \frac{5\times4}{2}(-2x)^2$

$= 1 - 10x + 40x^2$

4 (b) Find the first two non-zero terms in the expansion of

$$(1-2x)^5 + (1+5x)^2$$

in ascending powers of x.

[2 marks]

$(1+5x)^2 = 1 + 10x + 25x^2$

$2 + 65x^2$

4 (c) Hence, use an appropriate value of x to obtain an approximation for $0.998^5 + 1.005^2$

[2 marks]

$1 - 2x = 0.998 \qquad 2x = 0.002$

$x = 0.001$

$2 + 65(0.001^2)$

$= 2.000065$

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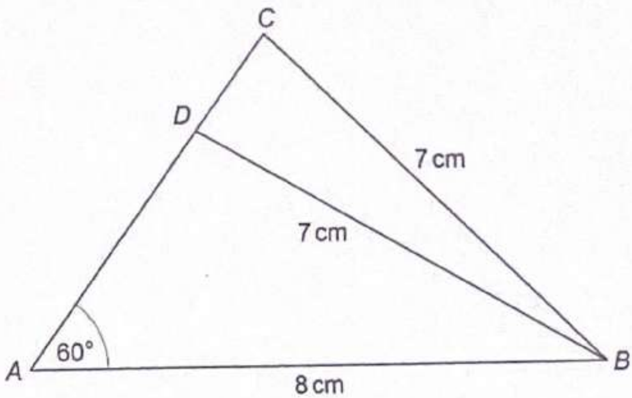
5 ABC is a triangle. The point D lies on AC.

$AB = 8\,\text{cm}$, $BC = BD = 7\,\text{cm}$ and angle $A = 60°$ as shown in the diagram.



5 (a) Using the cosine rule, find the length of AC.

[3 marks]

$a^2 = b^2 + c^2 - 2bc\cos A$.

$7^2 = 8^2 + c^2 - 2(8)c\cos(60)$

$49 = 64 + c^2 - 8c$

$c^2 - 8c + 15 = 0$

$c = 3$ or 5

$AC = 5\,\text{cm}$.

5 (b) Hence, state the length of AD.

[1 mark]

$AD = 3\,\text{cm}$

Do not write outside the box

6 Find the solution to

$$5^{(2x+4)} = 9$$

giving your answer in the form $a + \log_5 b$, where a and b are integers.

[3 marks]

~~log~~

$2x + 4 = \log_5 9$

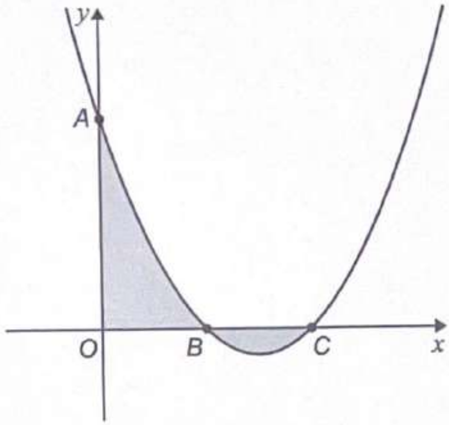
$2x + 4 = \log_5 (3^2)$

$2x + 4 = 2\log_5 3$

$x = 2 + \log_5 3$

Do not write outside the box

7 The diagram below shows the graph of the curve that has equation $y = x^2 - 3x + 2$ along with two shaded regions.



7 (a) State the coordinates of the points A, B and C.

[2 marks]

$y = (x-2)(x-1)$ $x = 2$ $x = 1$

C B A

$(2, 0)$ $(1, 0)$ $(0, 2)$

7 (b) Katy is asked by her teacher to find the total area of the two shaded regions.

Katy uses her calculator to find $\int_0^2 (x^2 - 3x + 2)\,dx$ and gets the answer $\frac{2}{3}$

Katy's teacher says that her answer is incorrect.

7 (b) (i) Show that the total area of the two shaded regions is 1

Fully justify your answer.

[5 marks]

1. integrate $O \rightarrow B$

$\int_0^1 x^2 - 3x + 2$

$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x \right]_0^1 = \frac{5}{6}$

Do not write outside the box

2. integrate $B \rightarrow C$

$$\int_1^2 x^2 \mp 3x + 2$$

$$\left[\frac{1}{3}x^3 + -\frac{3}{2}x^2 + 2x\right]_1^2 = -\frac{1}{6}$$

$$\frac{5}{6} - -\frac{1}{6} = \underline{1}$$

7 (b) (ii) Explain why Katy's method was not valid. **[1 mark]**

The calculator treats the area between B and C as negative.

Do not write outside the box

8 It is given that $y = 3x - 5x^2$

Use differentiation from first principles to find an expression for $\frac{dy}{dx}$

[4 marks]

first principles $f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \left[\frac{3(x+h) - 5(x+h)^2 - (3x - 5x^2)}{h} \right]$

$\lim_{h \to 0} \left[\frac{3x + 3h - 5x^2 - 10xh - 5h^2 - 3x + 5x^2}{h} \right]$

$\lim_{h \to 0} \left[\frac{3h - 10xh - 5h^2}{h} \right]$

$\lim_{h \to 0} \left[3 - 10x - 5h \right]$

$\frac{dy}{dx} = 3 - 10x$

Do not write outside the box

9 (a) Express $n^3 - n$ as a product of three factors.

[1 mark]

$n(n^2 - 1)$

$n(n-1)(n+1)$

9 (b) Given that n is a positive integer, prove that $n^3 - n$ is a multiple of 6

[3 marks]

$(n-1)n(n+1)$ is the product of 3 consecutive integers

So one must be a multiple of 3

And at least one must be a multiple of 2

So the product has factors of 2 and 3

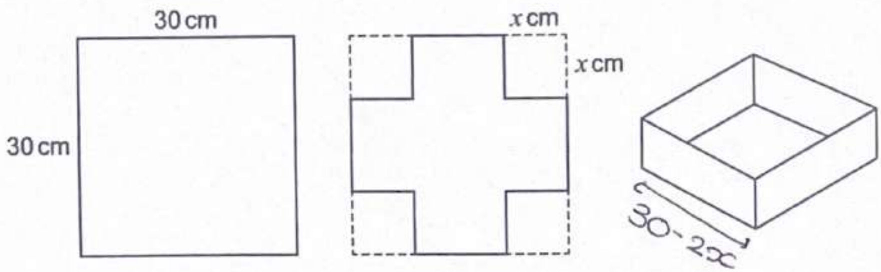
So is a multiple of $2 \times 3 = 6$

Turn over for the next question

10 A square sheet of metal has edges 30 cm long.

Four squares each with edge x cm, where $x < 15$, are removed from the corners of the sheet.

The four rectangular sections are bent upwards to form an open-topped box, as shown in the diagrams.



10 (a) Show that the capacity, $C\text{ cm}^3$, of the box is given by

$$C = 900x - 120x^2 + 4x^3$$

[2 marks]

30 − 2x

$x(30-2x)^2$

$= x(900 - 120x + 4x^2)$

$= 900x - 120x^2 + 4x^3$

10 (b) Find the maximum capacity of the box.

Fully justify your answer.

[7 marks]

$$\frac{dC}{dx} = 900 - 240x + 12x^2$$

at maximum, $\frac{dC}{dx} = 0$

$$0 = 900 - 240x + 12x^2$$

$$x = 5, 15$$

but $x < 15$, therefore $x = 5$

$$\frac{d^2C}{dx^2} = -240 + 24x$$

at $x = 5$, $-240 + 24(5) < 0$

$\therefore$ maximum at $x = 5$

$$C = 900(5) - 120(5^2) + 4(5^3)$$

$$= 2000 \text{ cm}^3$$

Do not write outside the box

11 A circle C has centre (0, 10) and radius $\sqrt{20}$

A line L has equation $y = mx$

11 (a) (i) Show that the x-coordinate of any point of intersection of L and C satisfies the equation

$$(1 + m^2)x^2 - 20mx + 80 = 0$$

[3 marks]

circle = $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-10)^2 = 20$

$x^2 + y^2 - 20y + 100 = 20$

$x^2 + y^2 - 20y + 80 = 0$

when $y = mx$

$x^2 + m^2x^2 - 20mx + 80 = 0$

$\therefore (1 + m^2)x^2 - 20mx + 80 = 0$

11 (a) (ii) Find the values of m for which the equation in part **(a)(i)** has equal roots.

[3 marks]

equal roots means $b^2 = 4ac$

$400m^2 = 4 \times (1 + m^2) \times 80$

$5m^2 = 4 \times (1 + m^2)$

$m^2 = 4$

$m = \pm 2$

Do not write outside the box

11 (b) Two lines are drawn from the origin which are tangents to C.

Find the coordinates of the points of contact between the tangents and C.

[4 marks]

when $m=2$ $\quad 5x^2 - 40x + 80 = 0$

$x = 4 \quad y = 8$

when $m=-2$ $\quad 5x^2 + 40x + 80 = 0$

$x = -4 \quad y = 8$

$(4, 8) \quad (-4, 8)$

Turn over for the next question

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Section B

Answer **all** questions in the spaces provided.

12 The table below shows the total monthly rainfall (in mm) in England and Wales in a sample of six years.

The sample of six years was taken from a data set covering every year from 1768 to 2018.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1768	109.2	129.1	12.8	85.6	46.1	148.7	121.9	91.6	136.8	119.4	142.5	103.6
1818	98.0	65.8	134.7	135.6	55.9	31.2	50.4	21.0	115.6	75.8	112.0	46.8
1868	99.9	62.2	71.1	61.4	36.7	16.5	20.0	106.7	90.2	95.6	61.4	185.6
1918	91.2	61.6	36.7	63.3	58.5	30.9	110.0	62.9	189.5	69.1	66.3	122.5
1968	85.8	47.6	59.5	68.8	78.7	94.0	107.8	72.2	148.1	99.0	69.6	84.2
2018	104.5	52.8	115.1	91.4	51.9	16.5	39.6	76.7	67.0	75.8	104.9	116.0

Deduce the sampling method **most likely** to have been used to collect this sample.

Circle your answer.

[1 mark]

Opportunity   Simple Random   Stratified   Systematic

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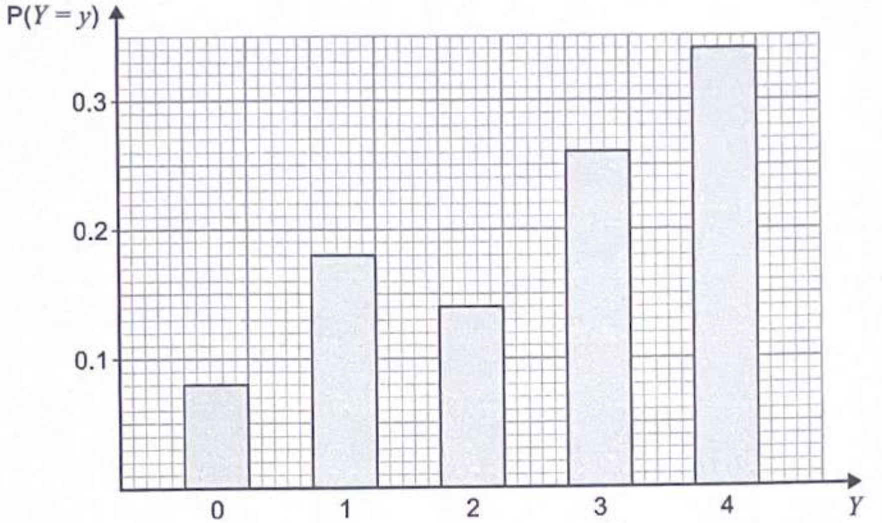
13 The diagram below shows the probability distribution for a discrete random variable Y.



Find $P(0 < Y \leq 3)$.

Circle your answer.

[1 mark]

0.40 0.42 0.58 0.66

Turn over for the next question

14 The random variable T follows a binomial distribution where

$$T \sim \mathrm{B}(16, 0.3)$$

The mean of T is denoted by μ.

14 (a) Find $\mathrm{P}(T \leq \mu)$.

[2 marks]

$\mu = np$

$\mu = 16 \times 0.3$

$\mu = 4.8$

$P(T \leq 4.8) = P(T \leq 4)$

$= 0.4499$

14 (b) Find the variance of T.

[1 mark]

$\text{Variance} = 16 \times 0.3 \times 0.7$

$= 3.36.$

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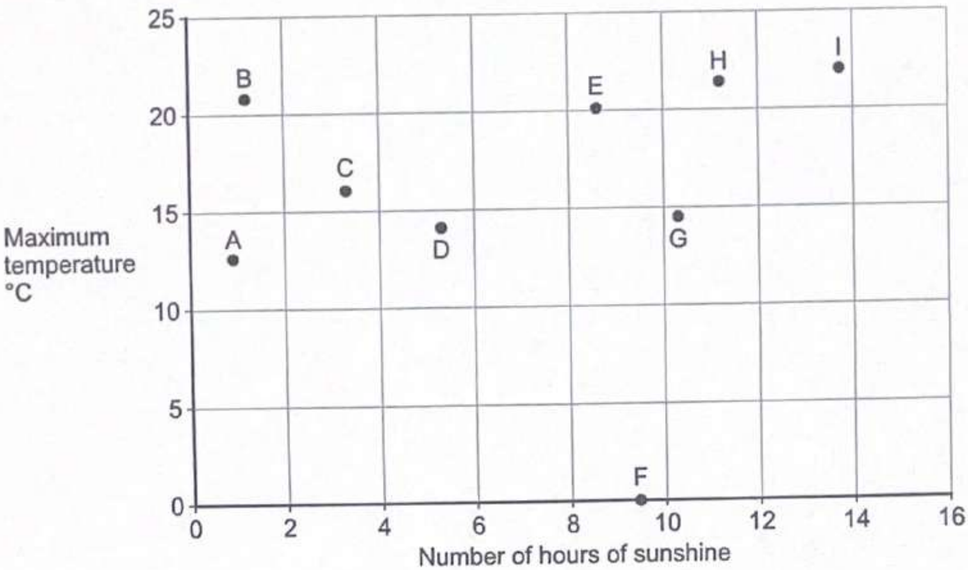
15 The number of hours of sunshine and the daily maximum temperature were recorded over a 9-day period in June at an English seaside town.

A scatter diagram representing the recorded data is shown below.



One of the points on the scatter diagram is an error.

15 (a) (i) Write down the letter that identifies this point.

[1 mark]

F

15 (a) (ii) Suggest one possible action that could be taken to deal with this error.

[1 mark]

Remove the point from the dataset

Do not write outside the box

15 (b) It is claimed that the scatter diagram proves that longer hours of sunshine cause higher maximum daily temperatures.

Comment on the validity of this claim.

[1 mark]

Invalid because correlation does not imply causation

Turn over for the next question

Do not write outside the box

16 An analysis was carried out using the Large Data Set to compare the CO_2 emissions (in g/km) from 2002 and 2016.

The summary statistics for the CO_2 emissions, X, for all cars registered as owned by either females or males is given in the table below.

	2002	**2016**
$\sum x$	207 901	142 103
Sample size	1215	1144

16 (a) Find the reduction in the mean of the CO_2 emissions in 2016 compared to the mean of the CO_2 emissions in 2002.

[2 marks]

Mean in 2002 = 171

mean in 2016 = 124

171 - 124 = 47

16 (b) It is claimed that the move to more electric and gas/petrol powered cars has caused the reduction in the mean CO_2 emissions found in part **(a)**.

Using your knowledge of the Large Data Set, state whether you agree with this claim.

Give a reason for your answer.

[1 mark]

~~[illegible]~~ disagree because there are very few electric car types in 2016, so they alone cannot have caused this reduction in the mean emissions.

Do not write outside the box

16 (c) There are 3827 data values in the Large Data Set.

It is claimed that the data in the table above must have been summarised incorrectly.

16 (c) (i) Explain why this claim is being made.

[1 mark]

this claim is being made because 1215 + 1144 =
2359 ≠ 3827

16 (c) (ii) State whether this claim is correct.

Give a reason for your answer.

[1 mark]

incorrect because the LDS catogory also has a
'company car' category

Turn over for the next question

Do not write outside the box

17 The number of toilets in each of a random sample of 200 properties from a town was recorded.

Four types of properties were included: terraced, semi-detached, detached and apartment.

The data is summarised in the table below.

	Number of toilets			
	One	**Two**	**Three**	
Terraced	20	10	4	34
Semi-Detached	18	50	16	84
Detached	12	10	8	30
Apartment	22	30	0	52
	72	100	28	200

One of the properties is selected at random.

A is the event 'the property has exactly two toilets'.

B is the event 'the property is detached'.

17 (a) (i) Find P(A).

[1 mark]

$P(A) = \frac{100}{200} = 0.5$

17 (a) (ii) Find P($A' \cap B$).

[1 mark]

$P(\text{not}A + B)$ $\cancel{B} = \frac{20}{200} = 0.1$

Do not write outside the box

17 (a) (iii) Find $P(A \cup B)$.

[2 marks]

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{100 + 30 - 10}{200} = \frac{120}{200}$

$= 0.6$

17 (b) Determine whether events A and B are independent.

Fully justify your answer.

[2 marks]

if $P(A) \times P(B) = P(A \cap B)$ independent

$P(A) \times P(B) = \frac{100}{200} \times \frac{30}{200} = 0.075$

$P(A \cap B) = \frac{10}{200} = 0.05$

$0.075 \neq 0.05$ not independent

17 (c) Using the table, write down two events, **other than event A and event B**, which are mutually exclusive.

[1 mark]

Event 1 property has 1 toilet

Event 2 property has 3 toilets

Turn over for the next question

18 It is known from previous data that 14% of the visitors to a particular cookery website are under 30 years of age.

To encourage more visitors under 30 years of age a large advertising campaign took place to target this age group.

To test whether the campaign was effective, a sample of 60 visitors to the website was selected. It was found that 15 of the visitors were under 30 years of age.

18 (a) Explain why a one-tailed hypothesis test should be used to decide whether the sample provides evidence that the campaign was effective.

[1 mark]

If the campaign is effective, the proportion of under 30s visitors will be greater than 14%

So a one-tailed test is required

18 (b) Carry out the hypothesis test at the 5% level of significance to investigate whether the sample provides evidence that the proportion of visitors under 30 years of age has increased.

[5 marks]

X is no. visitors under 30 on the website

$H_0 : p = 0.14$

$H_1 : p > 0.14$

Under H_0: $X \sim B(60, 0.14)$

$P(X \geq 15) = 1 - P(X \leq 14)$

$= 1 - 0.98351...$

$= 0.01649$

$= 0.0165$

As $0.0165 < 0.05$

Reject H_0

There is sufficient evidence to suggest that the advertising campaign has been effective

18 (c) State one necessary assumption about the sample for the distribution used in part **(b)** to be valid.

[1 mark]

The sample would need to be a random sample.

END OF QUESTIONS