AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname Lotfi-Baker

Forename(s) Nada
www.onlinetutors.xyz

Candidate signature

I declare this is my own work.

A-level
MATHEMATICS
Paper 2

Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
TOTAL	

Do not write outside the box

Section A

Answer **all** questions in the spaces provided.

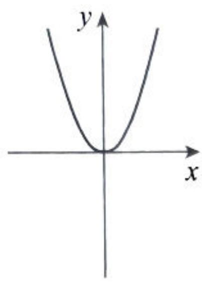
1 Four possible sketches of $y = ax^2 + bx + c$ are shown below.

Given $b^2 - 4ac = 0$ and a, b and c are non-zero constants, which sketch is the only one that could possibly be correct?

Tick (✓) **one** box.

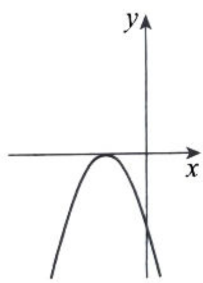
[1 mark]

A



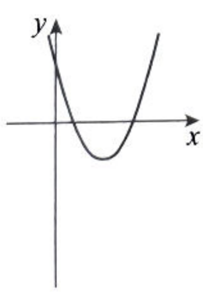
☐

B



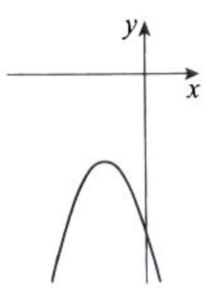
☑

C



☐

D



☐

2 A curve has equation $y = \mathrm{f}(x)$

The curve has a point of inflection at $x = 7$

It is given that $\mathrm{f}'(7) = a$ and $\mathrm{f}''(7) = b$, where a and b are real numbers.

Identify which one of the statements below must be true.

Circle your answer.

[1 mark]

$\mathrm{f}'(7) \neq 0$ $\quad$ $\mathrm{f}'(7) = 0$ $\quad$ $\mathrm{f}''(7) \neq 0$ $\quad$ $\mathrm{f}''(7) = 0$

3 A sequence is defined by

$$u_1 = a \text{ and } u_{n+1} = -1 \times u_n$$

Find $\sum_{n=1}^{95} u_n$

Circle your answer.

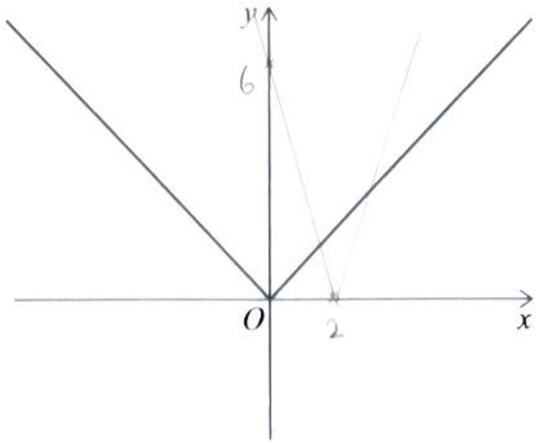
[1 mark]

$-a$ $\quad$ 0 $\quad$ a $\quad$ $95a$

Turn over for the next question

4 **Figure 1** shows the graph of $y = |2x|$

Figure 1



4 (a) On **Figure 1** add a sketch of the graph of

$$y = |3x - 6|$$

[2 marks]

4 (b) Find the coordinates of the points of intersection of the two graphs.

Fully justify your answer.

[4 marks]

$|2x| = |3x - 6|$

(1) $2x = 3x - 6$ $\quad x = 6$

(2) $-2x = 3x - 6$ $\quad x = 1.2$

$(6, 12)$ $\quad (1.2, 2.4)$

Turn over for the next question

Turn over ▶

Do not write outside the box

5 Express

$$\frac{5(x-3)}{(2x-11)(4-3x)}$$

in the form

$$\frac{A}{(2x-11)} + \frac{B}{(4-3x)}$$

where A and B are integers.

[3 marks]

$5(x-3) = A(4-3x) + B(2x-11)$

$5x - 15 = 4A - 3Ax + 2Bx - 11B$

$5 = -3A + 2B$

$-15 = 4A - 11B$

$A = -1 \quad B = 1$

$\frac{-1}{(2x-11)} + \frac{1}{(4-3x)}$

Do not write outside the box

6 Show that the solution of the equation

$$5^x = 3^{x+4}$$

can be written as

$$x = \frac{\ln 81}{\ln 5 - \ln 3}$$

Fully justify your answer.

[4 marks]

$\ln 5^x = \ln 3^{x+4}$

$x \ln 5 = (x+4)\ln 3$

$x\ln 5 - x\ln 3 = 4\ln 3$

$x(\ln 5 - \ln 3) = 4\ln 3$

$x(\ln 5 - \ln 3) = \ln 81$

$x = \dfrac{\ln 81}{\ln 5 - \ln 3}$

Turn over for the next question

7 A circle has equation

$$x^2 + y^2 - 6x - 8y = p$$

7 (a) (i) State the coordinates of the centre of the circle.

[1 mark]

$(3, 4)$

7 (a) (ii) Find the radius of the circle in terms of p.

[3 marks]

$x^2 + y^2 = 6x + 8y + p$

$x^2 + y^2 - 6x - 8y - p = 0$

$(x-3)^2 + (y-4)^2 - 9 - 16 - p = 0$

$(x-3)^2 + (y-4)^2 = 25 + p$

Radius $= \sqrt{25 + p}$

7 (b) The circle intersects the coordinate axes at exactly three points.

Find the **two** possible values of p.

[4 marks]

~~At x = 0 (x-3)² + (y-4)² = r²~~

intersects at 3 points = either passes through the origin or touches one of the axis

passes through origin $\sqrt{25+p} = 5$ $p = 0$

just touches the x axis $\sqrt{25+p} = 4$

$p = -9$.

Do not write outside the box

Turn over for the next question

8 Kai is proving that $n^3 - n$ is a multiple of 3 for all positive integer values of n.

Kai begins a proof by exhaustion.

Step 1		$n^3 - n = n(n^2 - 1)$
Step 2	When $n = 3m$, where m is a non-negative integer	$n^3 - n = 3m(9m^2 - 1)$ which is a multiple of 3
Step 3	When $n = 3m + 1$,	$n^3 - n = (3m + 1)((3m + 1)^2 - 1)$
Step 4		$= (3m + 1)(9m^2)$ $= 3(3m + 1)(3m^2)$ which is a multiple of 3
Step 5	Therefore $n^3 - n$ is a multiple of 3 for all positive integer values of n	

8 (a) Explain the two mistakes that Kai has made after Step 3.

[2 marks]

- Kai has not expanded the bracrets correctly
- kai has not considered numbers of the form $3m + 2$

8 (b) Correct Kai's argument from Step 4 onwards.

[4 marks]

Step 4: $= (3m+1)(9m^2 + 6m + 1 - 1)$

$= (3m+1)(9m^2 + 6m)$

$= 3(3m+1)(3m^2 + 2m)$

which is a multiple of 3

PTO

Do not write outside the box

Step 3: when $n = 3m+2$

$n^3 - n = (3m+2)((3m+2)^2 - 1)$

$= (3m+2)(9m^2 + 12m + 3)$

$= 3(3m+2)(3m^2 + 4m + 1)$

which is a multiple of 3.

$n^3 - n$ is always a multiple of 3 for all positive integer values of n.

Turn over for the next question

Do not write outside the box

9 A robotic arm which is attached to a flat surface at the origin O, is used to draw a graphic design.

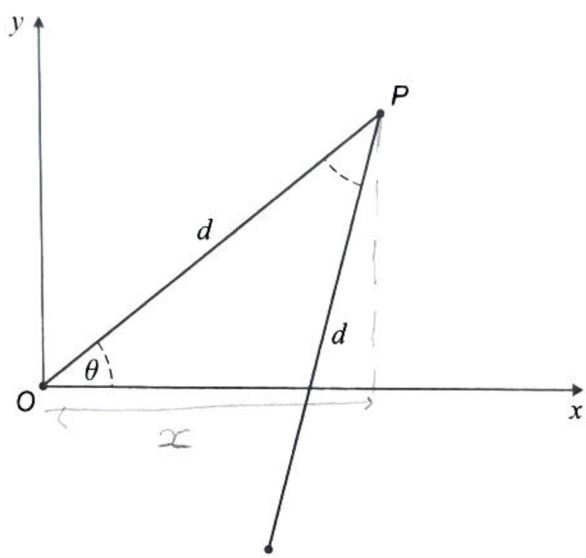
The arm is made from two rods OP and PQ, each of length d, which are joined at P.

A pen is attached to the arm at Q.

The coordinates of the pen are controlled by adjusting the angle OPQ and the angle θ between OP and the x-axis.

For this particular design the pen is made to move so that the two angles are always equal to each other with $0 \leq \theta \leq \frac{\pi}{2}$ as shown in **Figure 2**.

Figure 2



9 (a) Show that the x-coordinate of the pen can be modelled by the equation

$$x = d\left(\cos\theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$$

[2 marks]

QP makes angle 2θ with horizontal

$x = d\cos\theta - d\cos 2\theta$

$x = d\cos\theta - d\sin\left(\frac{\pi}{2} - 2\theta\right)$

$x = d\left(\cos\theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$

Do not write outside the box

9 (b) Hence, show that

$$x = d(1 + \cos\theta - 2\cos^2\theta)$$

[2 marks]

$x = d(\cos\theta - \cos 2\theta)$

$x = d(\cos\theta - (2\cos^2\theta - 1))$

$x = d(1 + \cos\theta - 2\cos^2\theta)$

9 (c) It can be shown that

$$x = \frac{9d}{8} - d\left(\cos\theta - \frac{1}{4}\right)^2$$

State the greatest possible value of x and the corresponding value of $\cos\theta$

[2 marks]

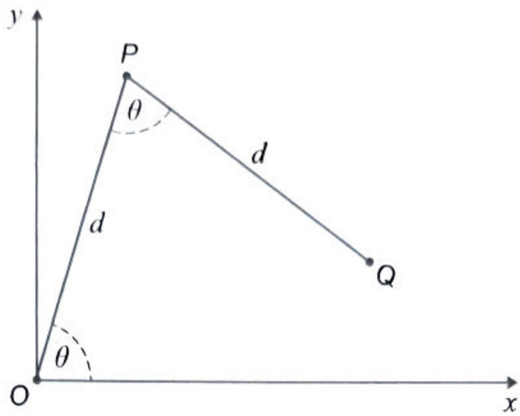
greast possible value $= \frac{9d}{8}$

at $\cos\theta = \frac{1}{4}$

Question 9 continues on the next page

9 (d) **Figure 3** below shows the arm when the x-coordinate is at its greatest possible value.

Figure 3



Find, in terms of d, the exact distance OQ.

[3 marks]

Using the cosine rule ...

$$OQ^2 = d^2 + d^2 - 2d^2 \cos\theta$$

$$= 2d^2 - 2d^2\left(\frac{1}{4}\right)$$

$$= \frac{3d^2}{2}$$

$$OQ = \frac{\sqrt{6}}{2}d$$

Turn over for the next question

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

10 The function h is defined by

$$h(x) = \frac{\sqrt{x}}{x-3}$$

where h has its maximum possible domain.

10 (a) Find the domain of h.

Give your answer using set notation.

[3 marks]

$\{ x : x \geq 0 , x \neq 3 \}$

10 (b) Alice correctly calculates

$$h(1) = -0.5 \text{ and } h(4) = 2$$

She then argues that since there is a change of sign there must be a value of x in the interval $1 < x < 4$ that gives $h(x) = 0$

Explain the error in Alice's argument.

[2 marks]

h (x) is not continuous at $x = 3$ meaning that a change of sign between $x = 1$ and 4 does not impy a root.

Do not write outside the box

10 (c) By considering any turning points of h, determine whether h has an inverse function.

Fully justify your answer.

[6 marks]

$$h(x) = \frac{\sqrt{x}}{x-3}$$

$$\frac{\frac{d}{dx}[\sqrt{x}](x-3) - \sqrt{x}\left(\frac{d}{dx}[x-3]\right)}{(x-3)^2}$$

$$= \frac{\frac{1}{2}x^{-1/2}(x-3) - x^{1/2}}{(x-3)^2}$$

$h'(x) = 0$ $\quad \frac{\frac{1}{2}x^{-1/2}(x-3) - x^{1/2}}{(x-3)^2} = 0$

$$\frac{1}{2}x^{-1/2}(x-3) - x^{1/2} = 0$$

$$\frac{(x-3)}{2\sqrt{x}} - \sqrt{x} = 0$$

$$x - 3 - 2x = 0$$

$$x = -3$$

$x = -3$ is not in the domain of h hence the function has no turning points

$h(x) > 0$ for $x > 3$

$h(x) < 0$ for $x < 3$

$\therefore$ function is one to one and has an inverse

Turn over for Section B

Do not write outside the box

Section B

Answer **all** questions in the spaces provided.

11 A particle's displacement, r metres, with respect to time, t seconds, is defined by the equation

$$r = 3e^{0.5t}$$

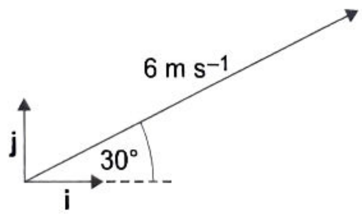
Find an expression for the velocity, $v\,\text{m s}^{-1}$, of the particle at time t seconds.

Circle your answer.

[1 mark]

$v = 1.5e^{0.5t}$ $\quad$ $v = 6e^{0.5t}$ $\quad$ $v = 1.5te^{0.5t}$ $\quad$ $v = 6te^{0.5t}$

12 A particle has a speed of $6\,\text{m s}^{-1}$ in a direction relative to unit vectors **i** and **j** as shown in the diagram below.



The velocity of this particle can be expressed as a vector $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\,\text{m s}^{-1}$

Find the correct expression for v_2

Circle your answer.

[1 mark]

$v_2 = 6\cos 30°$ $\quad$ $v_2 = 6\sin 30°$ $\quad$ $v_2 = -6\sin 30°$ $\quad$ $v_2 = -6\cos 30°$

Do not write outside the box

13 A vehicle, of total mass 1200 kg, is travelling along a straight, horizontal road at a constant speed of $13\,\text{m}\,\text{s}^{-1}$

This vehicle begins to accelerate at a constant rate.

After 40 metres it reaches a speed of $17\,\text{m}\,\text{s}^{-1}$

Find the resultant force acting on the vehicle during the period of acceleration.

[3 marks]

1200

$v^2 = u^2 + 2as$

$17^2 = 13^2 + 2a(40)$

$289 = 169 + 80a$

$a = 1.5\,\text{ms}^{-2}$

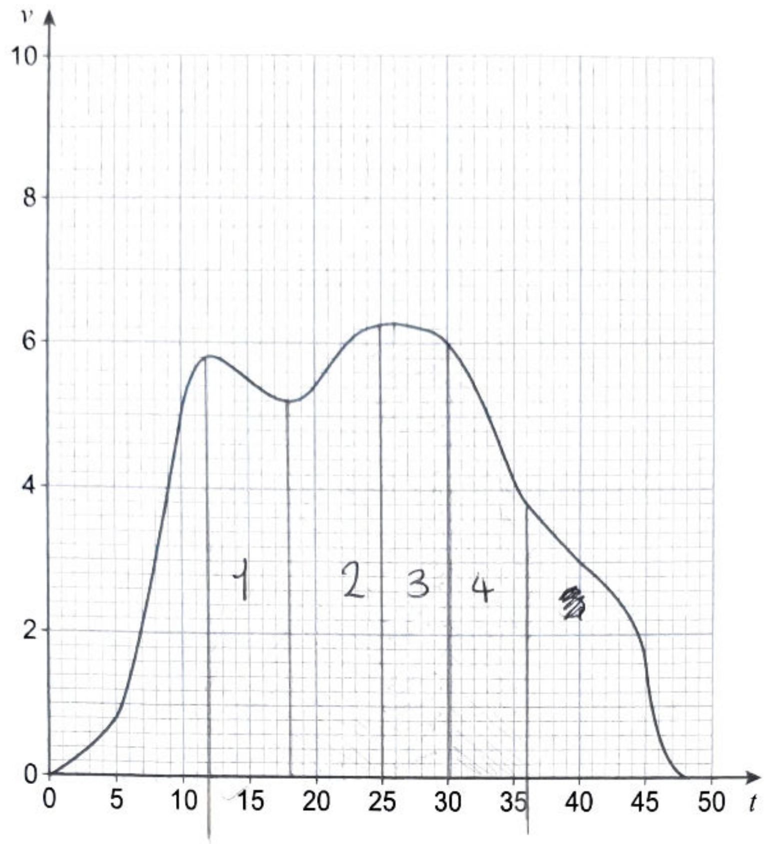
$F = ma$

$F = 1200 \times 1.5$

$= 1800\,\text{N}$

Turn over for the next question

14 A motorised scooter is travelling along a straight path with velocity $v\,\text{m}\,\text{s}^{-1}$ over time t seconds as shown by the following graph.



Noosha says that, in the period $\mathbf{12 \le t \le 36}$, the scooter travels approximately 130 metres.

Determine if Noosha is correct, showing clearly any calculations you have used.

[4 marks]

Distance = area under the curve

∘ split into 4 trapezia

1 → $6 \times \frac{1}{2}(5.8 + 5.2) = 33$

2 → $7 \times \frac{1}{2}(5.2 + 6.2) = 39.9$

3 → $5 \times \frac{1}{2}(6.2 + 6) = 30.5$

4 → $6 \times \frac{1}{2}(6 + 3.8) = 30.5$

Total $= 33 + 39.9 + 30.5 + 30.5$

$= 132.8 \approx 130$

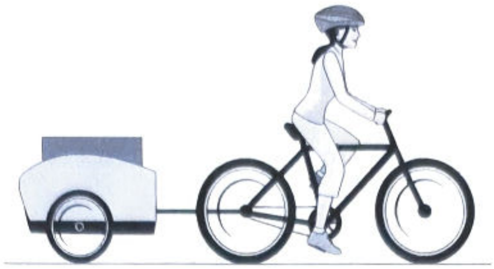
Noosha's estimate was reasonable

Do not write outside the box

Turn over for the next question

15 A cyclist is towing a trailer behind her bicycle.

She is riding along a straight, horizontal path at a constant speed.



A tension of T newtons acts on the connecting rod between the bicycle and the trailer.

The cyclist is causing a constant driving force of 40 N to be applied whilst pedalling forwards on her bicycle.

The constant resistance force acting on the trailer is 12 N

15 (a) State the value of T giving a clear reason for your answer.

[2 marks]

$T = 12$

constant speed means forces on the traile are in equilibrium

15 (b) State one assumption you have made in reaching your answer to part **(a)**.

[1 mark]

The rod remains horizontal

Do not write outside the box

15 (c) Find the external resistance force acting on the cyclist and her bicycle.

[2 marks]

$40 = T + R$

$40 + 12 + R$

$R = 28$ N

Turn over for the next question

Do not write outside the box

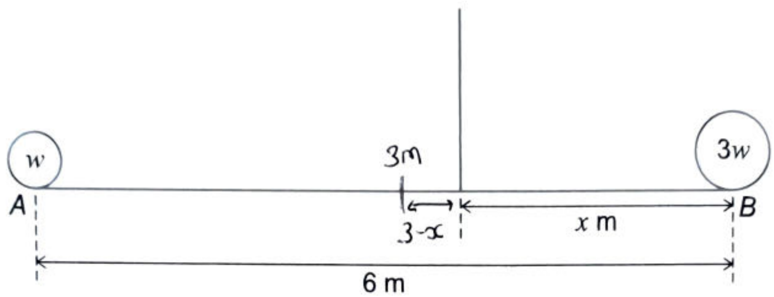
16 A straight uniform rod, AB, has length 6 m and mass 0.2 kg

A particle of weight w newtons is fixed at A.

A second particle of weight $3w$ newtons is fixed at B.

The rod is suspended by a string from a point x metres from B.

The rod rests in equilibrium with AB horizontal and the string hanging vertically as shown in the diagram below.



Show that

$$x = \frac{3w + 0.3g}{2w + 0.1g}$$

[4 marks]

Taking moments:

$3wx = (3-x)0.2g + (6-x)w$

$3wx = 0.6g - 0.2gx + 6w - wx$

$4wx + 0.2gx = 6w + 0.6g$

$\therefore x = \dfrac{3w + 0.3g}{2w + 0.1g}$

Do not write outside the box

Turn over for the next question

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

17 A ball is released from a great height so that it falls vertically downwards towards the surface of the Earth.

17 (a) Using a simple model, Andy predicts that the velocity of the ball, exactly 2 seconds after being released from rest, is $2g\,\mathrm{m\,s^{-1}}$

Show how Andy has obtained his prediction.

[2 marks]

$v = u + at$

$u = 0 \quad t = 2 \quad a = g$

$v = 0 + (g \times 2)$

$= 2g\mathrm{ms}^{-1}$

17 (b) Using a refined model, Amy predicts that the ball's acceleration, $a\,\mathrm{m\,s^{-2}}$, at time t seconds after being released from rest is

$$a = g - 0.1v$$

where $v\,\mathrm{m\,s^{-1}}$ is the velocity of the ball at time t seconds.

Find an expression for v in terms of t.

[7 marks]

$a = \frac{dv}{dt}$

$\frac{dv}{dt} = g - 0.1v \quad \int \frac{dv}{dt} = \int g - 0.1v\, dt$

$\int \frac{1}{g - 0.1v}\, dv = \int dt$

$-10\ln(g - 0.1v) = t + c$

at $t = 0$, $v = 0 \quad -10\ln g = c$

PTO

Do not write outside the box

$-10\ln(g-0.1v) = t - 10\ln g$

$-10\ln \frac{g-0.1v}{g} = ge^{-0.1t}$

$v = 10g(1-e^{-0.1t})$

17 (c) Comment on the value of v for the two models as t becomes large.

[2 marks]

Andy's → The velocity keeps increasing

Amy's → The velocity approaches an upper limit

Turn over for the next question

18 Two particles, P and Q, are projected at the same time from a fixed point X, on the ground, so that they travel in the same vertical plane.

P is projected at an acute angle $\theta°$ to the horizontal, with speed $u\,\mathrm{m\,s^{-1}}$

Q is projected at an acute angle $2\theta°$ to the horizontal, with speed $2u\,\mathrm{m\,s^{-1}}$

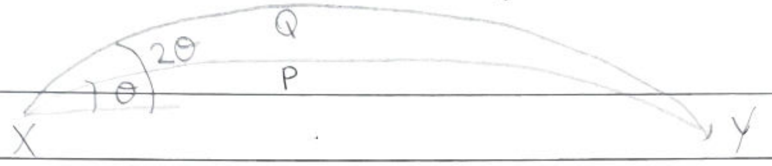
Both particles land back on the ground at exactly the same point, Y.

Resistance forces to motion may be ignored.

18 (a) Show that

$$\cos 2\theta = \frac{1}{8}$$

[6 marks]



$s = vt - \frac{1}{2}at^2$

vertical: $0 = t_p \sin\theta - \frac{1}{2} g t_p^2$

① $t_p = \dfrac{2u\sin\theta}{g}$

② $t_q = \dfrac{4u\sin 2\theta}{g}$

~~both~~ horizontal distances are equal so:

$t_p u\cos\theta = t_q 2u\cos 2\theta$

substitute ① + ② in

$\dfrac{2u^2\sin\theta\cos\theta}{g} = \dfrac{8u^2\sin 2\theta\cos 2\theta}{g}$

$2\sin\theta\cos\theta = 8\sin 2\theta\cos 2\theta$

$\frac{1}{8} = \cos 2\theta$

18 (b) P takes a total of 0.4 seconds to travel from X to Y.

Find the time taken by Q to travel from X to Y.

[4 marks]

$\cos\theta = 3/4$

$t_p \cos\theta = t_q 2\cos 2\theta$

$\cos\theta = 3/4, \quad \cos 2\theta = 1/8 \quad t_p = 0.4$

$0.4 \times 3/4 = t_q \times 2 \times 1/8$

$t_q = 1.2$ seconds

18 (c) State one modelling assumption you have chosen to make in this question.

[1 mark]

X and Y are the same height

Do not write outside the box

19 Two skaters, Jo and Amba, are separately skating across a smooth, horizontal surface of ice.

Both are moving in the same direction, so that their paths are straight and are parallel to each other.

Jo is moving with constant velocity $(2.8\mathbf{i} + 9.6\mathbf{j})\,\mathrm{m\,s^{-1}}$

At time $t = 0$ seconds Amba is at position $(2\mathbf{i} - 7\mathbf{j})$ metres and is moving with a constant speed of $8\,\mathrm{m\,s^{-1}}$

19 (a) (i) Explain why Amba's velocity must be in the form $k(2.8\mathbf{i} + 9.6\mathbf{j})\,\mathrm{m\,s^{-1}}$, where k is a constant.

[1 mark]

When 2 vectors are parallel, one is a scalar multiple of the other.

19 (a) (ii) Verify that $k = 0.8$

[1 mark]

Amba's velocity = $0.8(2.8i + 9.6j)$

Aba's speed = $\sqrt{2.24^2 + 7.68^2}$

$= 8\ \mathrm{ms^{-1}}$

19 (b) Find the position vector of Amba when $t = 4$

[3 marks]

$s = ut$

$s = 4 \times \begin{bmatrix} 2.24 \\ 7.68 \end{bmatrix} = \begin{bmatrix} 8.96 \\ 30.72 \end{bmatrix}$

$r = \begin{bmatrix} 2 \\ -7 \end{bmatrix} + \begin{bmatrix} 8.96 \\ 30.72 \end{bmatrix}$

$r = \begin{bmatrix} 10.96 \\ 23.72 \end{bmatrix}$ metres

19 (c) At **both** $t = 0$ and $t = 4$ there is a distance of 5 metres between Jo and Amba's positions.

Determine the shortest distance between their two parallel lines of motion.

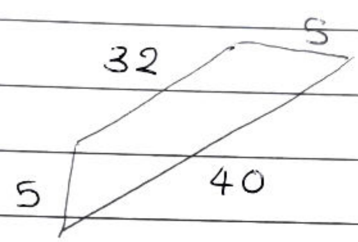
Fully justify your answer.

[5 marks]

Jo's speed $= \sqrt{2.8^2 + 9.6^2} = 10\text{ms}^{-1}$

Jo's distance $= 4 \times 10 = 40\text{m}$

Amba's distance $= 8 \times 4 = 32\text{m}$



$\sqrt{5^2 - \left[\frac{40-32}{2}\right]^2} = 3\text{m}$

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	Additional page, if required. Write the question numbers in the left-hand margin.

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**