SUVAT using gravity

Model Answers 1

Time Allowed: **40 minutes**

Score: **/33**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

Model Answer Key

Red = Answer
- This is what you need to write to get the mark
- Each bullet point represents 1 mark

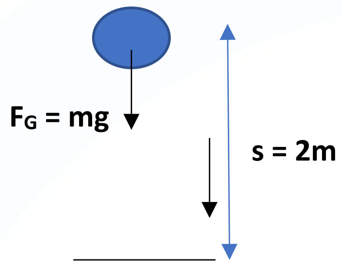
Blue = Explanation
- This is here to help you understand the answer
- You DON'T need to write this to get the marks

1. A ball of mass 0.3 kg is released from rest at a point which is 2 m above horizontal ground. The ball moves freely under gravity. After striking the ground, the ball rebounds vertically and rises to a maximum height of 1.5 m above the ground, before falling to the ground again. The ball is modelled as a particle.

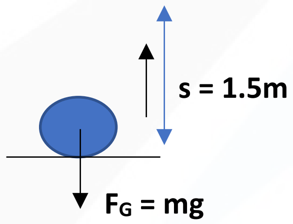
(a) Find the speed of the ball at the instant before it strikes the ground for the first time.

(2)

1.



2.



We use the following equation:

$$v^2 = u^2 + 2as$$

where v = final velocity (m/s)

u = initial velocity (m/s)

a = acceleration

s = displacement

By convention, we set the downwards direction as positive.

In this case, the initial velocity is 0. (in the moment the ball starts to fall)

Additionally, the ball falls freely under gravity, so the acceleration (a) is equivalent with the gravitational acceleration (g = 9.8m/s^2)

⇨ $v = \sqrt{4g}$

⇨ $v = \sqrt{4 \times 9.8\ m/s^2}$

v = 6.26 m/s

(b) Find the speed of the ball at the instant after it rebounds from the ground for the first time.

(2)

We use the same equation as in a)

$v^2 = u^2 + 2as$

When the ball rebounds from the ground, the final velocity will be equal to 0.

(The moment it hits the ground.)

The ball is also acting under gravity, so the acceleration (a) is the gravitational

acceleration ($g = 9.8\ m/s^2$)

⇨ $0 = u^2 - 3g$

⇨ $u = \sqrt{3g}$

⇨ $u = 5.42\ m/s$

(e) Find the time between the instant when the ball is released and the instant when it strikes the ground for the second time.

(4)

We use the following equation:

$v = u + at$

Where t = time (seconds)

a = acceleration

v = the final velocity (m/s)

u = the initial velocity (m/s)

In this case, the acceleration of the ball is equivalent to the gravitation

acceleration ($g = 9.8\ m/s^2$.

The final velocity will be associated with the time when it strikes the ground for

the second time (t_1) and the initial velocity will be associated with the time when

it is released (t_2)

⇨ $v = gt_1$ and $u = gt_2$

⇨ $t_1 = \frac{\sqrt{4g}}{g}$ and $t_2 = \frac{\sqrt{3g}}{g}$

⇨ $t_1 = 0.64$ s and $t_2 = 0.55$ s

Total time = $t_1 + 2\,t_2 = 0.64$ s + 2 x 0.55 s = **1.74 seconds**

(t_2 is considered twice because the ball falls twice and rebounds once)

(Total 8 marks)

2. At time $t = 0$, a particle is projected vertically upwards with speed u from a point A. The particle moves freely under gravity. At time T the particle is at its maximum height H above A.

(a) Find T in terms of u and g.

(2)

It always helps to draw a picture of what is happening:

The maximum height is achieved when the final velocity is v = 0.

We can use the following equation:

v = u + at

Where t = time (seconds)

a = acceleration

v = the final velocity (m/s)

u = the initial velocity (m/s)

In this case, the acceleration of the particle is g = 9.8 m/s^2 downwards. We will take upwards as positive, so this acceleration is negative.

$$0 = u + (-g)T$$

$$T = \frac{u}{g}$$

(b) Show that $H = \dfrac{u^2}{2g}$ **(2)**

We can use the following equation:

$$s = ut + \frac{1}{2}at^2$$

where t = time (seconds)
u = initial velocity (m/s)
a = acceleration
s = displacement (m)

In this case, the displacement s at time T is represented by the height H. Again, the acceleration of the particle is (g = 9.8 m/s^2).

$$S = uT + \frac{1}{2}(-g)T^2$$

$$T = \frac{u}{g}$$

$$H = \frac{u^2}{g} - \frac{1}{2}(-g)\left(\frac{u^2}{g^2}\right)$$

$$H = \frac{u^2}{2g}$$

The point A is at a height $3H$ above the ground.

(c) Find, in terms of T, the total time from the instant of projection to the instant when the particle hits the ground.

(4)

We can answer this question by finding out how long it takes the particle to reach the ground from its maximum height, and then adding that to the initial time worked out in part a).

$$s = ut + \frac{1}{2}at^2$$

$$-4H = 0 - \frac{1}{2}gt_{drop}^2$$

$$t_{drop} = \sqrt{\frac{8H}{g}}$$

Now using $H = \frac{u^2}{2g}$

$$t_{drop} = \sqrt{\frac{4u^2}{g^2}}$$

$$t_{drop} = \frac{2u}{g} = 2T$$

Therefore the total time is:

$$T_{\text{total}} = T + 2T$$
$$T_{\text{total}} = 3T$$

(Total 8 marks)

3. A small stone is projected vertically upwards from a point O with a speed of $19.6\,\text{m s}^{-1}$.

Modelling the stone as a particle moving freely under gravity,

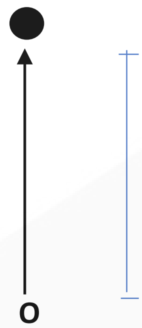
(a) find the greatest height above O reached by the stone,

(2)

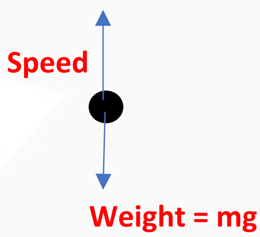
Speed of the stone: v = 19.6 m/s

By convention, we fix the upwards direction as positive.

The stone is projected vertically above point O. At the highest point, the velocity of the stone will be equal to 0.



The forces acting on the stone are:



We use the following equation:

$\text{Velocity}^2 = \text{speed}^2 + 2\,gh$

where h is the greatest height

g is the gravitational acceleration; $g = -9.8\ \text{m/s}^2$ (the acceleration acts downwards so the value needs to be negative)

⇨ $0^2 = 19.6^2 + 2\,.\,(-9.8).h$

⇨ $h = \frac{-19.6^2}{-19.6}$

⇨ **h = 19.6 m**

(b) find the length of time for which the stone is more than 14.7 m above *O*. **(5)**

$s > 14.7$ m where s is the displacement above O.

We use the equation:

$$s = ut + \frac{1}{2}at^2$$

g is the gravitational acceleration; $g = -9.8\ m/s^2$ (the acceleration acts downwards so the value needs to be negative)

$$14.7m = 19.6t - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 - 19.6t + 14.7 = 0$$

$$4.9t^2 - 19.6t + 14.7 = 0$$

$$t^2 - 4t + 3 = 0$$

$$\Leftrightarrow (t-1)(t-3) = 0$$

t = either 1 s or 3 s

Since the stone is above 14.7m between t=1 and t=3 the time for which it is above this height is

time = 3 - 1 = 2 seconds

(Total 7 marks)

4. A man throws a tennis ball into the air so that, at the instant when the ball leaves his hand, the ball is 2 m above the ground and is moving vertically upwards with speed $9\,\text{m s}^{-1}$

The motion of the ball is modelled as that of a particle moving freely under gravity and the acceleration due to gravity is modelled as being of constant magnitude $10\,\text{m s}^{-2}$

The ball hits the ground T seconds after leaving the man's hand.

Using the model, find the value of T. **(4)**

The initial velocity is 9 and it is released two metres above the ground.

s=0 at the release point (when t=0) and s=-2 at the ground as it is 2m below the man's hand.

Also, acceleration due to gravity is taken to be $10m^{-2}$, therefore the following SUVAT equation will allow us to find the time at which it hits the ground:

$$s = ut + \frac{1}{2}aT^2$$

$$-2 = 9T + \frac{1}{2}(-10)T^2$$

$$5T^2 - 9T - 2 = 0$$

$$(5T + 1)(T - 2) = 0$$

Ignoring the negative root for T, gives:

$$\boldsymbol{T = 2}$$

(Total 4 marks)

5. A stone is thrown vertically upwards with speed 16 m s^{-1} from a point h metres above the ground. The stone hits the ground 4 s later. Find

(a) the value of h,

(3)

The equation we can use is, considering vertical motion as positive.

$s = ut + \frac{1}{2} at^2$

where t = time, a = acceleration, u = initial velocity, s = displacement.

We need to work out the displacement after 4 seconds:

$h = 16 \times 4 + \frac{1}{2} \times 9.8 \times 4^2$

h= 14.4m

(*b*) the speed of the stone as it hits the ground. **(3)**

We can use the equation:

$v = u + at$

where v is the final velocity when it hits the ground and u is the initial velocity. t is the time and a is the acceleration.

$v = 16 + (-9.8) \times 4$

v = 23.2 m/s (ignore sign as it is speed)

(Total 6 marks)